Homework 5 is due April 17.

1. The stress-energy tensor for a perfect fluid is

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu + p g_{\mu\nu}, \tag{1}$$

where ρ and p are the energy density and pressure in the rest frame of the fluid, and u_μ is the 4-velocity of the fluid.

Derive the Euler equation

$$(p+\rho)\nabla_{\vec{u}}\vec{u} = -\nabla p - \vec{u}\nabla_{\vec{u}} p. \tag{2}$$

2. Show that a rocket ship that crosses inside the horizon of a Schwarzschild black hole will reach $r = 0$ in a proper time $\tau < \pi M$, no matter how the engines are fired.

Hint: use Schwarzschild coordinates

3. Two observers in two rockets are hovering above a Schwarzschild black hole of mass M. They hover at a fixed radius R such that (R/2GM -1)^1/2 e^(R/4M) = 1/2. Note R is then about 2.16M. The fist observer leaves his position at t=0 and travels into the black hole on a straight line in a Kruskal diagram until destroyed in the singularity at the point where the singularity crosses the line u=0 line. The other observer hovers at R.
a) On a Kruskal diagram, sketch the world lines of the 2 observers.
b) Is the observer who goes into the black hole following a timeline worldliness?
c) What is the latest Schwarzschild time after the first observer departs that the other observer can send a a light signal that will reach the first before being destroyed in the singularity?

4. Paul is in a circular orbit around a spherical nonspinning neutron star at r=6M. Peter is fired from a cannon, radially, from the surface of the neutron star (which has radius less than 6M), at less than escape velocity. Peter passes near Paul on his way out, reaches some maximum radius, and then passes near Paul on the way back down. Paul has completed 10 orbits during this time. Peter and Paul compare their clocks both times they pass. How much time has elapsed on Paul's clock? On Peter's clock?

5. Rough draft of your paper - 8-10 pages single column. This page count includes figures.